

NAG Toolbox for MATLAB

f08kp

1 Purpose

f08kp computes the singular value decomposition (SVD) of a complex m by n matrix A , optionally computing the left and/or right singular vectors.

2 Syntax

```
[a, s, u, vt, info] = f08kp(jobu, jobvt, a, 'm', m, 'n', n)
```

3 Description

The SVD is written as

$$A = U\Sigma V^H,$$

where Σ is an m by n matrix which is zero except for its $\min(m, n)$ diagonal elements, U is an m by m unitary matrix, and V is an n by n unitary matrix. The diagonal elements of Σ are the singular values of A ; they are real and nonnegative, and are returned in descending order. The first $\min(m, n)$ columns of U and V are the left and right singular vectors of A .

Note that the function returns V^H , not V .

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobu** – string

Specifies options for computing all or part of the matrix U .

jobu = 'A'

All m columns of U are returned in array **u**.

jobu = 'S'

The first $\min(m, n)$ columns of U (the left singular vectors) are returned in the array **u**.

jobu = 'O'

The first $\min(m, n)$ columns of U (the left singular vectors) are overwritten on the array **a**.

jobu = 'N'

No columns of U (no left singular vectors) are computed.

Constraint: **jobu** = 'A', 'S', 'O' or 'N'.

2: **jobvt** – string

Specifies options for computing all or part of the matrix V^H .

jobvt = 'A'

All n rows of V^H are returned in the array **vt**.

jobvt = 'S'

The first $\min(m, n)$ rows of V^H (the right singular vectors) are returned in the array **vt**.

jobvt = 'O'

The first $\min(m, n)$ rows of V^H (the right singular vectors) are overwritten on the array **a**.

jobvt = 'N'

No rows of V^H (no right singular vectors) are computed.

jobvt and **jobu** cannot both be 'O'.

Constraint: **jobvt** = 'A', 'S', 'O' or 'N'.

3: **a(lda,*)** – complex array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A .

5.2 Optional Input Parameters

1: **m** – int32 scalar

Default: The first dimension of the array **a**.

m , the number of rows of the matrix A .

Constraint: $\mathbf{m} \geq 0$.

2: **n** – int32 scalar

Default: The second dimension of the array **a**.

n , the number of columns of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldu, ldvt, work, lwork, rwork

5.4 Output Parameters

1: **a(lda,*)** – complex array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobu** = 'O', **a** is overwritten with the first $\min(m, n)$ columns of U (the left singular vectors, stored column-wise).

If **jobvt** = 'O', **a** is overwritten with the first $\min(m, n)$ rows of V^H (the right singular vectors, stored row-wise).

If **jobu** \neq 'O' and **jobvt** \neq 'O', the contents of **a** are destroyed.

2: **s(*)** – **double array**

Note: the dimension of the array **s** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The singular values of A , sorted so that $\mathbf{s}(i) \geq \mathbf{s}(i+1)$.

3: **u(ldu,*)** – **complex array**

The first dimension, **ldu**, of the array **u** must satisfy

if **jobu** = 'S' or 'A', **ldu** $\geq \max(1, \mathbf{n})$;
ldu ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, ucol)$, where *ucol* is the number of columns of U requested

If **jobu** = 'A', **u** contains the m by m unitary matrix U .

If **jobu** = 'S', **u** contains the first $\min(m, n)$ columns of U (the left singular vectors, stored column-wise).

If **jobu** = 'N' or 'O', **u** is not referenced.

4: **vt(ldvt,*)** – **complex array**

The first dimension, **ldvt**, of the array **vt** must satisfy

if **jobvt** = 'A', **ldvt** $\geq \max(1, \mathbf{n})$;
 if **jobvt** = 'S', **ldvt** $\geq \max(1, \min(\mathbf{m}, \mathbf{n}))$;
ldvt ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobvt** = 'A', **vt** contains the n by n unitary matrix V^H .

If **jobvt** = 'S', **vt** contains the first $\min(m, n)$ rows of V^H (the right singular vectors, stored row-wise).

If **jobvt** = 'N' or 'O', **vt** is not referenced.

5: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobu**, 2: **jobvt**, 3: **m**, 4: **n**, 5: **a**, 6: **lda**, 7: **s**, 8: **u**, 9: **ldu**, 10: **vt**, 11: **ldvt**, 12: **work**, 13: **lwork**, 14: **rwork**, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If f08kp did not converge, **info** specifies how many superdiagonals of an intermediate bidiagonal form did not converge to zero. See the description of **rwork** above for details.

7 Accuracy

The computed singular value decomposition is nearly the exact singular value decomposition for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. In addition, the computed singular vectors are nearly orthogonal to working precision. See Section 4.9 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is approximately proportional to mn^2 when $m > n$ and m^2n otherwise.

The singular values are returned in descending order.

The real analogue of this function is f08kb.

9 Example

```

jobu = 'Overwrite A by U';
jobvt = 'Singular vectors (V)';
a = [complex(0.96, -0.8100000000000001), complex(-0.03, +0.96), complex(-
0.91, +2.06), complex(-0.05, +0.41);
      complex(-0.98, +1.98), complex(-1.2, +0.19), complex(-0.66, +0.42),
      ...
      complex(-0.8100000000000001, +0.5600000000000001);
      complex(0.62, -0.46), complex(1.01, +0.02), complex(0.63, -0.17),
complex(-1.11, +0.6);
      complex(-0.37, +0.38), complex(0.19, -0.54), complex(-0.98, -0.36),
complex(0.22, -0.2);
      complex(0.83, +0.51), complex(0.2, +0.01), complex(-0.17, -0.46),
complex(1.47, +1.59);
      complex(1.08, -0.28), complex(0.2, -0.12), complex(-
0.07000000000000001, +1.23), complex(0.26, +0.26)];
[aOut, s, u, vt, info] = f08kp(jobu, jobvt, a)

```

```

aOut =
-0.5634 + 0.0016i    0.2687 + 0.2749i   -0.2451 - 0.4657i   -0.3787 -
0.2987i
 0.1205 - 0.6108i    0.2909 - 0.1085i   -0.4329 + 0.1758i    0.0182 +
0.0437i
-0.0816 + 0.1613i    0.1660 - 0.3885i    0.4667 - 0.3821i    0.0800 +
0.2276i
 0.1441 - 0.1532i   -0.1984 + 0.1737i    0.0034 - 0.1555i   -0.2608 +
0.5382i
-0.2487 - 0.0926i   -0.6253 - 0.3304i   -0.2643 + 0.0194i   -0.1002 -
0.0140i
-0.3758 + 0.0793i    0.0307 + 0.0816i   -0.1266 - 0.1747i    0.4175 +
0.4058i
s =
 3.9994
 3.0003
 1.9944
 0.9995
u =
-7.9551e+198 - 1.3029e-54i
vt =
-0.6971             -0.0867 - 0.3548i    0.0560 - 0.5400i   -0.1878 -
0.2253i
-0.2403             -0.0725 + 0.2336i    0.2477 + 0.5291i   -0.7026 -
0.2177i
 0.5123             0.3030 + 0.1735i   -0.0678 - 0.5162i   -0.4418 -
0.3864i

```

0.4403	-0.5294 - 0.6361i	0.3027 + 0.0346i	-0.1667 -
0.0258i			
info =			
0			
